# Primary school children's model building processes by the example of Fermi questions

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#### Abstract

The development of modelling skills is considered an important goal of mathematics education, also at primary school. In this context, manifold potentials are ascribed to so-called Fermi questions. We therefore emphasise the importance of further exploring how primary school pupils actually handle these kinds of problems.

In this article, with reference to a case study, we develop a suggestion for a descriptive progression model of work processes that take place when dealing with Fermi questions. This model combines aspects of both modelling and problem solving in a fruitful way.

#### 1. Introduction

Model building processes play a major role in current discussions and research on mathematics education (e.g. Blum, Galbraith, Henn, & Niss, 2007; Kaiser, Blomhøj, Sriraman, 2006; Sriraman, Kaiser, Blomhøj, 2006), and adequate competencies of students are an important goal of mathematics classes, as stipulated for example in German education standards or in the context of the PISA-study's *Mathematical Literacy*-concept (OECD, 2004). However, the reader may ask herself why this topic is picked up within the context of this volume on problem solving.

We believe that there are several similarities between problem solving and modelling (also e.g. Niss, Blum, & Galbraith, 2007), and that both approaches can be combined in a fruitful way (e.g. Greefrath, 2008). This becomes even more obvious when taking a closer look at the processes involved, or when comparing different process modellings as depicted in existing literature.

Describing model building processes is usually done by means of a cycle; Figure 1 shows a typical version by Blum (1985). This cycle begins with the actual situation which must then be structured, simplified and idealised by identifying relevant pieces of information with regard to the problem statement. Of course, the real model thus created retains a subjective touch, amongst others due to the individually available mathematical tools that, during the following step, can be used to develop a mathematical model that must fit to the real model. Here, different types of mathematisations are often possible. The formulation of a mathematical collows fre-

quently developed under consideration of one's own personal skills. In a final step, the mathematical results must be revised, or applied to the actual situation. Should they prove useless, models have to be amended and the cycle repeated.<sup>1</sup>

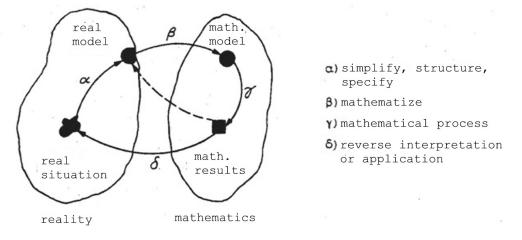


Figure 1: Cyclic model of modelling by Blum (1985)

If you compare the cyclic model of modelling with a progression model for problem solving – e.g. by Pólya (1971) – you will encounter some parallels that are schematically visualised in Figure 2 and according to which modelling processes can be understood as one specific type of problem solving.

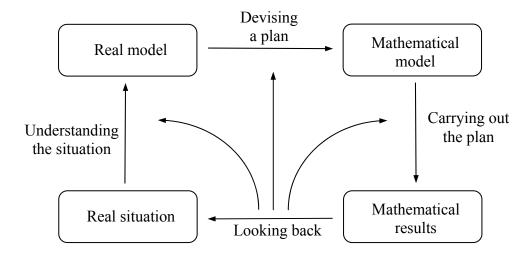


Figure 2: Modelling and problem solving

# 2. Theoretical background

According to German education standards, modelling is a general mathematical competence to be developed also at primary schools (Kultusministerkonferenz, 2005).

<sup>&</sup>lt;sup>1</sup> In newer models of the model building process, some elements are elaborated further. A differentiation is frequently made between the actual situation and its mental representation as the starting point of modelling (in a narrower sense) (e.g. Borromeo Ferri, 2006).

The following example can be found in an explanatory publication (Walther, van den Heuvel-Panhuizen, Granzer, & Köller, 2008) to illustrate this area of competence (p. 35):

	4000 pupils in 48 classes? Can that be
4000 Schüler in 48 Schulklassen Gevelsberg – Die Sommerferien neigen sich dem Ende zu. Die vielen Kinder, die zu Fuß zur Schule unterwegs sind, sind ein Zeichen, dass die 9 Schulen in Gevelsberg wieder ge- öffnet sind. Dieses Schuljahr sind es fast 4000 Schüler, die zusammen 48 Schulklassen besuchen. Für manche Schüler waren die Ferien viel zu kurz, aber die meisten freuen sich darauf, ein neues Schuljahr zu beginnen.	
50:11:55 1000:50=200 Nein, er gilt heine Klaine in der 80 King 4000 pupils in 48 classes	du sina
<i>Gevelsberg</i> - Summer holidays are coming to an end. number of children walking to school are a clear indi Gevelsberg's 9 schools have once again opened their do This year, almost 4000 pupils are enrolled in a total of For some children, the summer break was much too most of them are looking forward to the new school year	cation that ors. 48 classes. short, but
No, there are no classes of 80 pupils.	

Figure 3: Proposed modelling task

According to the authors, to solve this exercise, pupils must extract relevant information and neglect other data; translate a realistic problem into mathematical terms and develop a mathematical model; solve it inner-mathematically; and finally refer back to the initial situation (cf. Figure 1).

However, I think this task is a fairly simple one, as the pupil working on it only has to choose two out of three given figures (both of which are included in the heading of the newspaper article), and only make one calculation to come to a basic "yes" or "no" conclusion.

Contrary to this task, "good" modelling problems that require actual model building (and problem solving) can be described by attributes such as *realistic, data-based, complex, open, differentiating*, while these characteristics are not independent of each other and the fifth rather refers to the possible use of the problem in class (Henze, 2009). Model building processes thus consist less of neglecting empirical details until the "skeleton", which they created, can be translated into mathematical terms, but rather of a *structural extension* of the situation, of introducing new elements and (mathematical) relations (Schwarzkopf, 2006). Is it possible to successfully employ such "good" modelling problems, enabling authentic mathematical activity, already at primary school? One possibility to do this is seen in so-called Fermi questions<sup>2</sup> (Kaufmann, 2006; Peter-Koop, 2004, 2008; Wälti, 2005) which are problems providing no or insufficient information for a calculational solution and for which the goal is to define a suitable solution range through justifiable assumptions (Kaufmann, 2006).

The high potential that can be seen in Fermi questions when used in class makes it plausible to us to further examine the question in which way students actually deal with these kinds of challenges. However, the cycle in Figure 1 is hardly a suitable point of departure for developing a descriptive model of relevant work processes. For at primary school it is rarely the case that, when solving a modelling task, the factual context must temporarily be left completely (in the sense of the modelling cycle); that, so to speak, a "pure" inner-mathematical problem is created. One rather makes the effort to ensure that the children do not lose connection with the issues at hand even though they are busy with their calculations (Schwarzkopf, 2006).

The descriptive system by Möwes-Butschko, on the other hand, could prove more useful. Its focus is rather on individual processes and was employed by the author to describe the handling of "open realistic tasks" by primary school pupils. Figure 4 shows one example of the used exercises combining text and pictures (Möwes-Butschko, 2007).



How tall is the baby elephant?

Figure 4: An "open realistic task" used by Möwes-Butschko

Möwes-Butschko suggests the following categories to describe the pupils' work processes (without further explanation): *Orientation, Planning, Data collection, Data processing, Data securing, Argumentation, and Control.* 

### 3. Research question

Within the frame of a teaching experiment, Fermi questions (without pictures) were employed in two different Year 4 classes. The idea was to explore whether the pupils' work processes

 $<sup>^{2}</sup>$  Named after Enrico Fermi (1901 – 1954), a prominent nuclear physicist of the 20th century who repeatedly gave such exercises to his students. A typical example, ascribed to Fermi himself, is the question of the number of piano tuners in Chicago.

can be described by the categories suggested by Möwes-Butschko, and in how far a further differentiation of these categories seems appropriate. Another aim was to gain further indications regarding the frequently postulated potential of Fermi questions in modern mathematics education.

# 4. Implementation of the study

For the case study, one Year 4 class was chosen from each of two primary schools in the German federal state of Lower Saxony. The lessons were held shortly before the end of the primary schooling period, so that we could expect the pupils to possess the knowledge and competencies set by the curriculum.

Pupils in Class A were used to a very traditional teaching style regarding content and methodology and did not have any experience in working on modelling exercises. Class B was also not familiar with modelling exercises; they had, however, worked in groups before.

The experiment comprised a total of four lessons. During the first lesson, pupils were able to gather first experiences with Fermi questions, and possible approaches were discussed in class. To us, it was important that pupils understood and accepted that there is not one definite and exact solution to the problem, that you must frequently estimate, that there are several possible ways to solve the problem, and that various auxiliary tools may be used (research, expert interviews,...). A fixed problem solving scheme such as in Figure 1 was not discussed, however.

During the second and third lessons, the pupils dealt with the following Fermi questions: "How many worksheets do you complete during your time at school?" and "How much time of your life do you spend brushing your teeth?" Results and work processes were to be recorded on a group poster for the following presentation and discussion of results.

Two groups of pupils were chosen from each class and filmed while working on their first Fermi question. When choosing the group members, attention was paid to the fact that these pupils worked together well and could easily pick up a conversation. The performance level of individual pupils was not relevant.

Finally, during the fourth lesson, the pupils developed their own Fermi questions.

When selecting the Fermi questions, we wanted to make sure that most pupils would be able to handle the (obvious<sup>3</sup>) mathematical demands, and that the context of the questions would be clear to them. By addressing pupils individually ("How many worksheets do *you* complete

<sup>&</sup>lt;sup>3</sup> From the point of view of the persons designing the question.

during your time at school?"), they were encouraged to contribute by sharing their own experiences.

Data collection was crucial for the first question, while processing this data should then be a fairly simple exercise. To solve the second question, however, data collection played a minor role, while dealing with different time units was the trickier task to tackle for primary school pupils. Both questions demanded strong argumentation skills.<sup>4</sup>

In a first step, the videotaping was transcribed. For one group of pupils, the transcription was interpreted through qualitative content analysis by two reviewers independently of each other. The aim was to trace the work process of the *group*. The setup of this analysis was generally open and gave room for creating categories, whereas Möwes-Butschko's findings and basic considerations on modelling and problem solving served as the general points of departure. The devised categorisations were then compared, differences discussed and subsequently, joint decisions regarding their allocation were made. The following step was an open analysis of the other groups' work processes. Repeated rounds of analysis served to revise all categorisations and introduce refining sub-categories where suitable.

#### 5. Results

Due to the limited scope of this paper, we can only discuss the modelling processes of two groups of pupils in more detail. Yet, in order to give an idea of the broad spectrum that was to be observed, we will present two very different work processes, both from class B.

Two of the three boys in Team B1 will be attending "Gymnasium" (grammar school)<sup>5</sup> after the summer holidays, so their grades in mathematics are accordingly high. The third boy has received a recommendation to continue secondary schooling at "Realschule", yet, he is very good at mathematics. All three of them are rather reserved compared to the two girls and two boys in Team B2. One pupil in this group is repeating Year 4, and his grades in mathematics are average, like one other pupil's in his team. The other two generally perform very well.

The first team is looking for an answer to the question of how many worksheets have to be completed during time at school.

After the teacher had presented the question to the plenum, the pupils tried to gain their own

<sup>&</sup>lt;sup>4</sup> The following questions must be answered, amongst others: How many school years and subjects are taken into consideration? How does the number of work sheets change, respectively? Starting from which age to which final year, how many times daily and for how long do you brush your teeth?

<sup>&</sup>lt;sup>5</sup> In Germany, pupils are allocated to one of four types of secondary school depending on their performance and abilities shown in primary school: Gymnasium (8 years, leads up to university education), Realschule (6 years, intermediate level), Hauptschule (5-6 years, prepares for vocational training); Gesamtschule combines the three types in one institution.

understanding of the problem within their group and spontaneously gave their opinions (*Orientation – understanding the problem*<sup>6</sup>). Following an initial phase of cluelessness, the boys began to develop first ideas – it was suggested, amongst others, to go through all folders of previous school years and count the worksheets filed there –, which were not picked up again or elaborated further, though (*Orientation – first ideas*). They later came up with the idea to estimate the respective number of worksheets per subject per school year – an approach that seemed practicable to the pupils (*Planning*). They started making a list of the subjects taught at primary school (*Collecting data*). When estimating the number of worksheets per subject (*Collecting data, Estimations*), it partly came to longer discussions (*Collecting data, Discussion*), after which the total number for all 4 years of primary school still had to be calculated (*Processing data, Calculations*). A longer working phase thus resulted in a first answer to the initial problem statement.

In a following short conversation with the teacher, the pupils first of all reported on their approach and their current work progress (*Report to external party/Ask for help*). They also discussed whether secondary schools should be taken into consideration when answering the Fermi question. After the pupils had agreed on this, they commenced a further working phase very similar to the one just described. However, there were longer discussions regarding the type of secondary school to be examined (6 years of "Realschule" or 8 years of "Gymnasium", see footnote 5), and there was some insecurity regarding the list of subjects (number of foreign languages).

The pupils worked on the problem for a total of 32 minutes in a motivated manner (which very much surprised the teacher, who was at first skeptical). In the process, each of them was able to contribute his ideas, there were intensive discussions regarding the approach to be taken, the list of subjects and the respective number of worksheets. Phases of data collection and -processing were very extensive, calculative skills were intensively practiced during the latter ones.

Figure 5 shows the poster on which the pupils recorded their results, which they proudly presented at the end of the lesson. The high total number of 6160 worksheets may be surprising, but we believe it is also an indication of the impression pupils get of their lessons.

<sup>&</sup>lt;sup>6</sup> See following section on category system

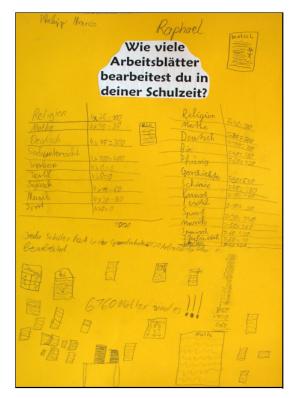


Figure 5: Poster designed by Team B1

For a more detailed assessment of the pupils' work progress, a more process oriented analysis could be valuable. The refined category system developed for this purpose will be presented in the following section.

#### Category system

Some of the categories which distinguish the work processes have already been mentioned in previous sections of this article. Overall, the observed (videotaped and transcribed) work processes can be described by a category system consisting of six "basic categories" of modelling, and further six categories including "interaction" as well as "further activities". For a more detailed description, we can define additional six sub-categories of modelling which further specify the basic categories.<sup>7</sup> It thus seems appropriate to differentiate between estimations, discussions and research within the basic category "collecting data", and between calculations, argumentations and control within "processing data", for example. The suggested differentiation of basic- and sub-categories enables, on the one hand, a very precise description of the observed processes. On the other hand, the variable degree of resolution also makes the category system applicable to other Fermi questions or modelling tasks.

<sup>&</sup>lt;sup>7</sup> It goes without saying that only certain combinations are useful.

The following overview names the categories, briefly explains them and, for some cases, gives additional illustrative examples.

Basic categories of modelling	Sub-categories of modelling		
Orientation – understanding the problem	Spontaneous solution	S	
	Calculations	С	
Planning	Estimations	Е	
Collecting data	Argumentation / Discussion	D	
Processing data	Control	0	
Reflection	Use of auxiliary material / Research	М	
Teacher-student interaction	<i>Further activities</i>		
Report to external party/Ask for help	Team work/Preparing poster/Secure c	lata	
Impulse	Context		
	No reference to the problem		
	Others		

# Basic categories of modelling

- Orientation – understanding the problem: It is about the basic understanding of the problem.

Number	Person	Statement	Category	Sub- category
3	Р	How many work sheets do you complete during your time at school? <b>**</b> No idea.		
4	М	Ohh that will be hard. [LAUGHING]		

- Orientation – first ideas: First ideas on how to solve the problem are expressed. However, these do not lead far and must ultimately be abandoned or modified.

8	Μ	### we should check up every ring binder,	
14	R	then we have round about 1000'	S

- Planning: A critical idea for solving the problem is found, or further proceedings are planned with foresight. Possibly, metacognitive approaches can be identified.

43 M I've got an idea: we can write down the school subjects,

- Collecting data: Data is collected, usually by applying everyday knowledge or making estimations, also by using auxiliary means or research.

	53	М	In mathematics we do not have so many,	D
5	54	R	Sure,	D
5	55	Р	Nope,	D

- Processing data: Collected data is processed. This takes place mainly by carrying out calculations. Justifying a certain calculation is also a constitutive part of data processing.

151	R	75 times 4 is erm*	C
152		5sec	C
153	Р	2 times 75-	C
154	R	300,** isn't it'	C

- Reflection: Pupils mainly reflects on the meaning of the results, reflection on the approach can also take place (while this must not be confused with control of individual calculations).

555	R	Man, a great many! 6160, if one thinks about,	
556	Р	Hm,	
557	R	Imagine you have to copy so many -	
558	Р	hohoho [LAUGHING]	
559	R	How expensive would it be'	

# Sub-categories of modelling

- Spontaneous solutions: Spontaneous solutions are very rough estimations of the solution or simply an imagined result that cannot be further explained.
- Calculations: Calculations are subordinated to data processing. Very simple auxiliary calculations and conversions of units are assigned to this category, just like complicated calculations of the final result.
- Estimations: Estimated values are allocated to this category as well as their possible justification, as latter usually refers to basic supporting knowledge which is linked to the estimations.

482	R	7 times 20* 140, music' because of all the songs,	Е
483	М	30' oh no, more,	Е
484	Р	40 round about,	Е
485	R	I'll write down 50,	Е

- Argumentation/Discussion: Processes of argumentation and discussion take place during many phases of the modelling process. In categorisation, they are highlighted only in processing and collecting data. In this context, data processing mainly implies discussing possible approaches; in data collection, it is above all about justifying the selection of data.
- Control: With regard to control, we also differentiate between data collecting and data processing. In former case, it is rather the completeness of selected data that is checked, in latter case the individual calculations.

285	R	Then only 3,	0
286	Р	Only 3, let's take Spanish-	0
287	R	Latin and Italien,	0

- Use of auxiliary material/Research: This can be understood as a sub-category of collecting data. Auxiliary means or tools vary according to the problem at hand. When conducting research, both, books as well as the Internet or other sources can be drawn upon.

## Teacher-student interaction

- Impulse: Impulses come from the teacher. During the time that her students are working on the problem, the teacher acts above all as a supervisor who gives support in case of questions or problems. Her comments and remarks that were made in these cases are called impulses.
- Report to external party/Ask for help: Students come into interaction with the teacher. They report on their current state of progress or search for assistance by asking questions.

# Further activities

- Team work/Prepare poster/Secure data: Processes in this category include all group work, the making of the poster and data backup.
- Context: This category points to statements made that lie within the context of the problem situation.
- No reference to the problem: This category summarises all statements made by students which bear no reference to the problem itself.
- Others: A rest category includes all statements that are not categorisable, or events which cause an interruption of the process through external circumstances, such as a break.

We allocated Möwes-Butschko's category "Data securing" to the category "Team work/Preparing poster/Secure data"; this may be due to the nature of the assignment given to the pupils, who used their poster for all notes, calculations etc. from the beginning on. Phases of review (cf. Pólya's model) or metacognition could not be clearly identified – also because the remarks in question can be subject to a wide scope of interpretation. Therefore, they were not explicitly included in the category system.

### Analysis of the work processes

Within the limits of this paper we can thus only describe work processes with a low degree of resolution, i.e. we must neglect the sub-categories. If you determine the relative amount of

time Team B1 spent on the respective categories based on the pupils' verbalizations – which makes a certain "blurring" unavoidable – the result is the following diagram.

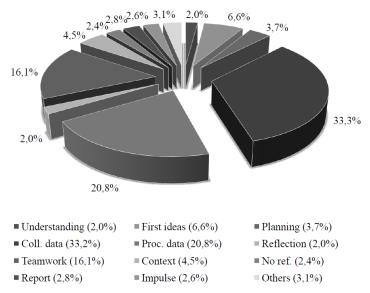


Figure 6: Percentage of time spent on each process category (Team B1)

This diagram confirms the impression we got during the observations: The pupils worked on the data very intensively, data collection and processing took a lot of time. It also becomes clear that, in general, the pupils handled the questions very concentrated and quite independently.

The progress of problem solving is pictured in the following graph. Every field represents one second and every row one minute; it should thus be read line by line from left to right. Categories are indicated by respective colour shades and hatchings.

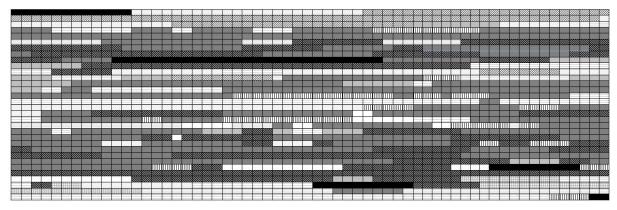


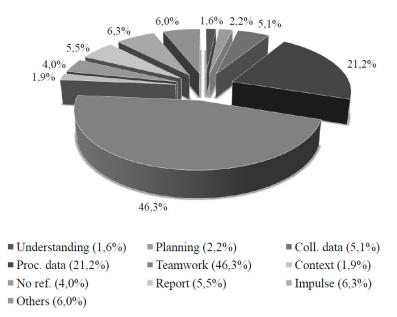
Figure 7: Progress of work processes (Team B1)

Obvious is a longer phase of orientation right at the beginning, during which pupils first of all tried to develop their own understanding of the question and developed first ideas. Towards the end of the process, a focus naturally laid on designing the poster. Before that, we could observe a phase of reflection, which "only" focused on the results, however, and not on one's

own work processes that had previously been carried out.

Apart from other details, the graph clearly illustrates that even work processes described by basic categories of modelling can *be repeated* and *without following a strict order* – for example, phases of orientation and planning took place again during a later stage of the progress.

The second team of pupils was working on the question of how much time of your life you spend brushing your teeth. The answer they came up with was 35 days. This relatively short time span can be explained by the fact that, while the pupils did discuss how often you brush your teeth per day, they made their final calculations taking into account only 2 minutes of brushing per day.



The relative amount of time spent on the respective categories is shown in Figure 8.

Figure 8: Percentage of time spent on each process category (Team B2)

As was to be expected, data collection was much less time consuming due to the nature of the question. As the required calculations are more difficult (different time units), the relative amount of time spent on data processing is even a little higher than with the first group. We could not observe a reflection of the results or of the process. Striking is the large percentage of the category "Team work/Preparing Poster/Secure data".

The following graph shows the progress of work processes.

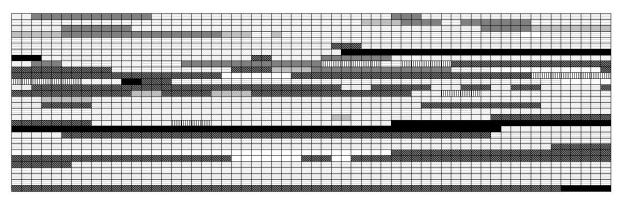


Figure 9: Progress of work processes (Team B2)

The graph suggests that it took the pupils a while in the beginning to initiate cooperation within the group, and also at a later stage, coordinating cooperation and poster design took up a lot of time. The group was quite obviously dependent on repeated impulses and assistance by the teacher.

# 6. Discussion

Also due to the observations made in the other groups, as well as in the two classes on the whole, using Fermi questions in class towards the end of primary school (i. e. Year 4) generally seems possible to us. They can encourage persistent involvement, initiate important work processes that go beyond the respective question at hand – especially regarding data handling – and simultaneously offer reasonable exercises for basic mathematical skills which are regarded as useful from a pupil's point of view. Metacognitive elements, however, could not be observed among the groups that took part in this study.

An important aspect from our point of view when using Fermi questions – as Team B2's work progress also suggests –, is that you should not distract too much from content-related aspects by focusing too much on the methodological design of the lesson.<sup>8</sup>

The constructed category system derived from content analyses combines aspects of mathematising non-mathematical questions or situations, and aspects of problem solving.

To us, it seems appropriate to describe model building processes among primary school pupils in more detail, also in their progress. This way, it is e.g. possible to trace varying demands of Fermi questions on pupils, or an according handling of these questions (e.g. extent of collecting data).

<sup>&</sup>lt;sup>8</sup> With regard to the aim of this study, we find it nonetheless appropriate to e.g. ask the pupils to design a poster which also shows considerations and interim results. This way, it is easier to understand the approaches and steps that pupils take to find a solution to the problem.

Due to the fact that this study was based on case studies, possible correlations between the progress of model building processes or the percentage of time spent on certain process categories, and the final success of the process, could not be explored. This could be an aim of a more extensive study in the future.

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